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Linear Acoustic Formulas for Calculation of Rotating Blade Noise

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A unified approach is used to derive many of the current formulas for calculation of discrete frequency noise of helicopter rotors and propellers. Both compact and noncompact results are derived. The noncompact results are based on the solution of Ffowcs Williams-Hawkings (FW-H) equation. The compact formulations are obtained as the limit of noncompact source results. In particular, the linearized acoustic theories of Hawkings and Lowson, Farassat, Hanson, Woan and Gregorek, Succi, and Jou are discussed in this paper. An interesting thickness noise formula by Isom and its extension by Ffowcs Williams are also presented.

Nomenclature

B	= number of blades
c	= speed of sound
$F(y;x,t)$	$=[f(y,\tau)]_{\text{ret}}=f(y,t-r/c)$
$F_i(\tau)$	= net force by a compact source on the medium
F_r	$=F_i\hat{r}_i$
$f(x,t)=0$	= equation of the surface of a body in motion
$\bar{f}$	= shaft frequency
g	$=\tau-t+r/c$
$H(\cdot)$	= Heaviside function
h	= the thickness function of a blade
i	$=\sqrt{-1}$
$J_n(\cdot)$	= Bessel function of first kind and nth order
k_D	= see Eq. (69)
L	= maximum dimension of Σ surface, $F=0$
L_{ki}	= net force on the medium by the kth segment of the blade (see the method of Succi)
l_i	= force/unit area on the medium
l_r	$=l_i\hat{r}_i$
M	$=R\omega/c$, local rotational Mach number
M_F	$=V_F/c$, Mach number of forward flight
M_n	$=v_n/c$
M_r	$=v_i\hat{r}_i/c$, local Mach number in the radiation direction
n	= harmonic number
n_i	$=\nabla f/\lvert\nabla f\rvert$, local outward unit normal on the body $f=0$
PF	= planform of the body $f=0$
p	= surface pressure p_a-p_0 on $f=0$
p'	= acoustic pressure
p_a	= absolute pressure
p_0	= ambient pressure of undisturbed medium
p'_T	= acoustic pressure due to thickness
p'_L	= acoustic pressure due to loading
p'_{Tn}	= the amplitude of nth harmonic of the thickness noise
p'_{Ln}	= the amplitude of nth harmonic of the loading noise
Q	= source distribution function
$\bar{Q}$	= Fourier transform of Q
Q_0	= net rate of mass injection from the surface $F=0$
R	= radial distance from rotation center
r	$=\lvert x-y\rvert$
$\hat{r}_i$	$=(x_i-y_i)/r$, unit radiation vector
$r_{\min}$	= minimum distance of the observer from the surface $F=0$
r_0	= observer distance from rotation center
ret	= retarded time
S	= surface area of the actual body $f=0$
$s(\tau)$	= source position vector
s'	= distance along the direction of helical velocity vector
T	= thrust or lift/unit area
$\bar{T}$	= period of the sound
t	= observer time
U	= local helical speed of a source
V_N	$=-(\partial F/\partial t)/\lvert\nabla F\rvert$, normal velocity of the surface $F=0$
V_F	= forward velocity of the propeller
v	$=(v_iv_i)^{1/2}$
v_i	= local velocity of the body or velocity of a compact source
v_n	$=-(\partial f/\partial t)/\lvert\nabla f\rvert$, local normal velocity of the body $f=0$
v_r	$=v_i\hat{r}_i$
x,x_i	= observer position
Y	= see Eq. (70)
y,y_i	= source position
β	= azimuthal angle of the source in nonrotating x-frame
Γ	= the curve of intersection $f=0$ and $g=0$
γ	= ratio of specific heats
Δ	= indicates the jump in a quantity as $\Delta p=(p)_{\text{lower}}-(P)_{\text{upper}}$
$\delta(\cdot)$	= the Dirac delta function
η,η_i	= the source position in a rotating frame fixed to the blade
θ	= angle between $\hat{r}_i$ and n_i
θ_0	= angle between propeller or rotor axis and the observer direction
Λ	$=[1+M_n^2-2M_n\cos\theta]^{1/2}$
λ	= transform variable [see Eq. (60)]
ρ_0	= density of the undisturbed medium
Σ_k	= summation over all segments of the blade
Σ	= surface area of $F=0$
σ	= twist angle
τ	= source time
Φ	= a function used in Eq. (2)
ϕ	= polar angle of the source in the frame fixed to the blade
Ψ	= volume inside $F=0$
ψ	= volume inside $f=0$
ψ_k	= volume inside kth segment of the blade (see the Method of Succi)
ω	= angular velocity of the blade
ω_D	= see Eq. (61)
$\square^2$	= the wave operator, D'Alembertian

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I. Introduction

FOR many years, there has been a need for accurate prediction of the noise of propellers and helicopter rotors. The increase in the number of general aviation aircraft and helicopters and the awareness of the public to environmental noise pollution have been driving forces for research on prediction techniques. A good review of the works in this field up to the early seventies is by Morfey.[1] Many other papers have been published since Morfey's review article, with new formulations for prediction of the noise of rotating blades. Most of these results are for noncompact sources. In general, one requires the use of a computer for obtaining the acoustic pressure signature or spectrum of the noise.

In this paper, a unified approach is used to derive many of the compact and noncompact formulas for the calculation of the discrete frequency noise of rotating blades. The approach is based on the Ffowcs Williams-Hawkings (FW-H) equation.[2] It is shown that many known results can easily be obtained by manipulating various forms of the solution of the FW-H equation. The methods presented here are not those used in the original derivations. For example, Hanson's results, discussed below, were originally based on the solution of an inhomogeneous wave equation as described by Morse and Ingard.[3] However, it was felt that the approach used in the present paper is more appropriate since it takes advantage of the operational properties of generalized functions and thus reduces the algebra.

In the following sections, the general noncompact and compact formulations are first presented. Then these equations are specialized as used by various researchers. In the interest of brevity, the frequency domain formula of Hanson and the acoustic formula of Jou are not derived in this paper but are presented for completeness.

II. General Acoustic Formulas

In this section, some general formulas for calculation of the sound from moving bodies will be derived. These will form the starting point for other formulas in the next section. Only linear acoustic results will be discussed.

Consider a body whose surface is described by the equation $f(\boldsymbol{x},t)=0$ where the $\boldsymbol{x}$-frame is fixed to the undisturbed medium and t is the time. The equation of the surface $f=0$ is defined such that $f>0$ outside the body and $f<0$ inside it. The FW-H equation without the quadrupole source term is

$$\Box^2 p' = \frac{1}{c^2}\frac{\partial^2 p'}{\partial t^2} - \nabla^2 p' = \frac{\partial}{\partial t}[\rho_0 v_n |\nabla f| \delta(f)] - \frac{\partial}{\partial x_i}[l_i |\nabla f| \delta(f)] \tag{1}$$

where p' is the acoustic pressure and ρ_0 and c are the density and speed of sound in the undisturbed medium, respectively. The local normal velocity of the body surface is $v_n = -(\partial f/\partial t)/|\nabla f|$ and l_i is the local force per unit area acting on the fluid at the surface of the body. The Dirac delta function is denoted by $\delta(f)$. The source terms on the right-hand side of Eq. (1) have become known as thickness noise and loading noise sources, respectively. The derivation of this equation is discussed in Refs. 2 and 4. The necessary mathematical background for the derivation of this equation and what follows here can be found in Ref. 4 and in the literature cited there. Note that the surface pressure term in l_i is actually the difference between the absolute surface pressure p_a and the undisturbed ambient pressure p_0. The solution of Eq. (1) in various forms will be presented below. First, however, the solution of the wave equation with a source distribution on a moving surface will be derived.

Solution of the Wave Equation with Surface Sources

Consider the wave equation

$$\Box^2\Phi = Q(\boldsymbol{x},t)\,|\nabla f|\,\delta(f) \tag{2}$$

This equation has a source term similar to the FW-H equation except for some time or space derivative on the right. It can be solved by Green's function method as follows. Green's function of the wave equation in unbounded space is $\delta(g)/4\pi r$, where $g=\tau-t+r/c$ and $r=|\boldsymbol{x}-\boldsymbol{y}|$. Here $\boldsymbol{x}$ and t are the observer position and time, $\boldsymbol{y}$ and τ are the source position and source time, respectively. The formal solution of Eq. (2) is

$$4\pi\Phi(\boldsymbol{x},t) = \int \frac{1}{r} Q(\boldsymbol{y},\tau)\,|\nabla f|\,\delta(f)\,\delta(g)\,\mathrm{d}\boldsymbol{y}\,\mathrm{d}\tau \tag{3}$$

The time integration is over $(-\infty,t)$ and the space integration is over the entire unbounded space.

A change of variable $\tau\rightarrow g$ and integration over g will result in

$$4\pi\Phi(\boldsymbol{x},t) = \int \frac{1}{r}[Q(\boldsymbol{y},\tau)\,|\nabla f|\,\delta(f)]_{\mathrm{ret}}\,\mathrm{d}\boldsymbol{y} \tag{4}$$

where the subscript ret stands for retarded time. Let the surface Σ be given by $F(\boldsymbol{y};\boldsymbol{x},t)=[f(\boldsymbol{y},\tau)]_{\mathrm{ret}}=f(\boldsymbol{y},t-r/c)=0$. Then, Eq. (4) can be written as

$$4\pi\Phi(\boldsymbol{x},t) = \int \frac{1}{r}[Q(\boldsymbol{y},\tau)\,|\nabla f|]_{\mathrm{ret}}\,\delta(F)\,\mathrm{d}\boldsymbol{y} \tag{5}$$

Now the following relations developed in Refs. 4 and 5 will be used to integrate the delta function:

$$\mathrm{d}\boldsymbol{y} = \frac{\mathrm{d}F\,\mathrm{d}\Sigma}{|\nabla F|} \tag{6}$$

$$|\nabla F| = [\,|\nabla f|\,\Lambda]_{\mathrm{ret}} \tag{7}$$

$$\Lambda^2 = 1 + M_n^2 - 2M_n\cos\theta \tag{8}$$

where $M_n = v_n/c$ and θ is the angle between ∇f and the radiation direction $\boldsymbol{r}=\boldsymbol{x}-\boldsymbol{y}$. Equation (5) can then be written as a surface integral over the Σ surface as follows:

$$4\pi\Phi(\boldsymbol{x},t) = \int_{F=0} \frac{1}{r}\left[\frac{Q(\boldsymbol{y},\tau)}{\Lambda}\right]_{\mathrm{ret}}\mathrm{d}\Sigma \tag{9}$$

This result is not surprising since the surface Σ is the locus of points on $f=0$ in space whose emitted signals arrive simultaneously to the point $\boldsymbol{x}$ at time t. This surface is the intersection of $f(\boldsymbol{y},\tau)=0$ and $g=\tau-t+r/c=0$ in $(\boldsymbol{y},\tau)$ space. Fortunately, the visualization of the Σ surface is very easy. Note that $(\boldsymbol{x},t)$ are kept fixed in the above derivations so that our attention will be focused on $\boldsymbol{y}$ and τ only. For a fixed source time $\tau<t$, the surface $g=0$ is a sphere with center at the observer and radius $r=c(t-\tau)$. As τ is varied from $-\infty$ to t, the radius of this sphere collapses to zero. This sphere intersects the surface $f(\boldsymbol{y},\tau)=0$ over some time interval during the collapsing process. The curve of intersection for a fixed τ will be denoted by Γ. The locus of these Γ curves in space is the surface $\Sigma{:}F=0$. It is shown in Ref. 5 that the following relations hold

$$\frac{\mathrm{d}\Sigma}{\Lambda} = \frac{c\,\mathrm{d}\Gamma\,\mathrm{d}\tau}{\sin\theta} \tag{10a}$$

$$= \frac{\mathrm{d}S}{|1-M_r|} \tag{10b}$$

where dS is the element of surface area of $f=0$ (assumed a rigid surface) and $M_r = v_i \hat{r}_i / c$. Here v_i is the local surface velocity on $f=0$ and $\hat{r}_i = (x_i - y_i)/r$ is the unit radiation vector. Note that in Eq. (10a), the body is not assumed rigid. Using the above relations, Eq. (9) can be written as

$$4\pi\Phi(x,t) = \int_{\substack{f=0\\g=0}} \frac{cQ(y,\tau)}{r\sin\theta}\,\mathrm{d}\Gamma\mathrm{d}\tau \tag{11a}$$

$$= \int_{f=0} \left[\frac{Q(y,\tau)}{r|1-M_r|}\right]_{\text{ret}} \mathrm{d}S \tag{11b}$$

The reason that r in Eq. (11b) is now evaluated at the retarded time is that, in the frame fixed to the body where the surface integral is evaluated, r is a function of the source time.

Returning to the FW-H equation, note that the derivatives appearing before the source terms are generalized derivatives.[4] They can be taken out of the integrals in the solution. In the following paragraphs, the analysis of this section will be utilized to develop formulas for the loading and thickness noise.

Noncompact Source Formulas

Loading Noise

The loading noise is described by the solution of the equation

$$\Box^2 p_L' = -\frac{\partial}{\partial x_i}[l_i |\nabla f| \delta(f)] \tag{12}$$

Using Eqs. (9) and (11), the solution of this equation can be written in the following forms:

$$4\pi p_L'(x,t) = -\frac{\partial}{\partial x_i}\int_{F=0} \frac{1}{r}\left[\frac{l_i}{\Lambda}\right]_{\text{ret}} \mathrm{d}\Sigma \tag{13a}$$

$$= -\frac{\partial}{\partial x_i}\int_{\substack{f=0\\g=0}} \frac{cl_i}{r\sin\theta}\,\mathrm{d}\Gamma\mathrm{d}\tau \tag{13b}$$

$$= -\frac{\partial}{\partial x_i}\int_{f=0}\left[\frac{l_i}{r|1-M_r|}\right]_{\text{ret}} \mathrm{d}S \tag{13c}$$

Equations (13a) and (13b) have not been utilized for noise calculations as they stand. Equation (13c) was employed by Hawkings and Lowson[6] to derive their acoustic formula presented in the next section. Some more useful formulas can be derived by converting the space derivative in the above equations to a time derivative.[4,5] The easiest method is to start with the formal solution of Eq. (12) before the integration of the delta functions $\delta(f)$ and $\delta(g)$. The relation

$$\frac{\partial}{\partial x_i}\left[\frac{\delta(g)}{4\pi r}\right] = -\frac{1}{c}\frac{\partial}{\partial t}\left[\frac{\hat{r}_i\delta(g)}{4\pi r}\right] - \frac{\hat{r}_i\delta(g)}{4\pi r^2} \tag{14}$$

is then used in the final step of conversion. The results for Eqs. (13) are

$$4\pi p_L'(x,t) = \frac{1}{c}\frac{\partial}{\partial t}\int_{F=0}\frac{1}{r}\left[\frac{l_r}{\Lambda}\right]_{\text{ret}}\mathrm{d}\Sigma + \int_{F=0}\frac{1}{r^2}\left[\frac{l_r}{\Lambda}\right]_{\text{ret}}\mathrm{d}\Sigma \tag{15a}$$

$$= \frac{\partial}{\partial t}\int_{\substack{f=0\\g=0}}\frac{l_r}{r\sin\theta}\,\mathrm{d}\Gamma\mathrm{d}\tau + \int_{\substack{f=0\\g=0}}\frac{cl_r}{r^2\sin\theta}\,\mathrm{d}\Gamma\mathrm{d}\tau \tag{15b}$$

$$= \frac{1}{c}\frac{\partial}{\partial t}\int_{f=0}\left[\frac{l_r}{r|1-M_r|}\right]_{\text{ret}}\mathrm{d}S + \int_{f=0}\left[\frac{l_r}{r^2|1-M_r|}\right]_{\text{ret}}\mathrm{d}S \tag{15c}$$

In these equations, $l_r = l_i\hat{r}_i$ is the force in the radiation direction per unit area. If the viscosity is neglected, one can write $l_r = p\cos\theta$ where p is the surface pressure defined as $p_a - p_0$. The absolute pressure on the body surface is denoted as p_a and p_0 is the ambient pressure of the undisturbed medium.

In this paper, the loading noise formula of Hanson[7] is derived using Eq. (15a). The starting point for the derivation of Hawkings and Lowson's result[6] is Eq. (15c). Farassat's method[8-10] is based on Eqs. (15b) and (15c). The method of Woan and Gregorek[11] is based on Eq. (15c).

By taking the time derivatives inside the integrals in Eq. (15), many other formulas can be derived which will not be presented because of their limited usefulness. The resulting equations can become very long and complicated.

Thickness Noise

The thickness noise is the solution of the equation

$$\Box^2 p_T' = \frac{\partial}{\partial t}[\rho_0 v_n |\nabla f| \delta(f)] \tag{16}$$

Because of the diversity of the forms of the solution, each separate derivation is numbered below.

1) Using the method discussed above for Eq. (2), the solution of this equation can be written as

$$4\pi p_T'(x,t) = \frac{\partial}{\partial t}\int_{F=0}\frac{1}{r}\left[\frac{\rho_0 v_n}{\Lambda}\right]_{\text{ret}}\mathrm{d}\Sigma \tag{17a}$$

$$= \frac{\partial}{\partial t}\int_{\substack{f=0\\g=0}}\frac{\rho_0 c v_n}{r\sin\theta}\,\mathrm{d}\Gamma\mathrm{d}\tau \tag{17b}$$

$$= \frac{\partial}{\partial t}\int_{f=0}\left[\frac{\rho_0 v_n}{r|1-M_r|}\right]_{\text{ret}}\mathrm{d}S \tag{17c}$$

These are the most useful forms for calculation of the thickness noise. In this paper, Hanson's time domain method is derived based on Eq. (17a).[7] Many other methods are based on Eqs. (17b) and (17c).

2) An entirely different form of the thickness noise formula was derived by Isom.[12] Because of its potential for numerical calculation and its interesting physical interpretation, this formula will be derived below. Note that Isom's original result was for a rotating blade with the observer in the far field. Farassat[13] showed that Isom's result was valid for any moving body and Ffowcs Williams extended the result to the near field. This extension was reported by Farassat.[14]

Let $H(f)$ denote the Heaviside function. Then, applying the wave operator to the function $[1-H(f)]\rho_0 c^2$, one obtains

$$\Box^2\{[1-H(f)]\rho_0 c^2\} = \frac{\partial}{\partial t}[\rho_0 v_n |\nabla f| \delta(f)] + \frac{\partial}{\partial x_i}[\rho_0 c^2 n_i |\nabla f| \delta(f)] \tag{18}$$

where $\mathbf{n} = \nabla f / |\nabla f|$ is the unit outward normal on the surface of the body $f=0$. Subtracting both sides of Eq. (16) from corresponding sides of Eq. (18) and using the relation $1 - H(f) = 0$ outside the body, the following differential equation for the thickness noise is obtained

$$\Box^2 p_T' = -\frac{\partial}{\partial x_i}[\rho_0 c^2 n_i |\nabla f| \delta(f)] \tag{19}$$

This means that the thickness noise is equivalent to the noise from a uniform pressure distribution of magnitude $\rho_0 c^2$ over the body $f=0$. In Isom's work, this interpretation of thickness noise can be deduced from his far field formula for rotating blades.

The solution of Eq. (19) is

$$4\pi p'_T(x,t) = -\frac{\partial}{\partial x_i}\int_{F=0}\frac{\rho_0 c^2}{r}\left[\frac{n_i}{\Lambda}\right]_{\text{ret}}\mathrm{d}\Sigma$$

$$=\frac{\partial}{\partial t}\int_{F=0}\frac{\rho_0 c}{r}\left[\frac{\cos\theta}{\Lambda}\right]_{\text{ret}}\mathrm{d}\Sigma+\int_{F=0}\frac{\rho_0 c^2}{r^2}\left[\frac{\cos\theta}{\Lambda}\right]_{\text{ret}}\mathrm{d}\Sigma \quad (20)$$

Four other forms of these relations can be obtained by using Eq. (10).[14] These are very unusual forms of thickness noise relations because of the absence of the term v_n.

This interpretation of the thickness noise can be used to write the FW-H equation in an unusual form. For an inviscid fluid, $l_i = (p_a - p_0)n_i$. Using the relation $\rho_0 c^2 = \gamma p_0$, where γ is the ratio of the specific heats, the FW-H equation can be written as

$$\square^2 p' = -\frac{\partial}{\partial x_i}\{[p_a + (\gamma-1)p_0]n_i|\nabla f|\delta(f)\} \quad (21)$$

The solution of this equation is

$$4\pi p'(x,t) = -\frac{\partial}{\partial x_i}\int_{F=0}\frac{1}{r}\left[\frac{\tilde{p}n_i}{\Lambda}\right]_{\text{ret}}\mathrm{d}\Sigma \quad (22a)$$

$$=\frac{1}{c}\frac{\partial}{\partial t}\int_{F=0}\frac{1}{r}\left[\frac{\tilde{p}\cos\theta}{\Lambda}\right]_{\text{ret}}\mathrm{d}\Sigma+\int_{F=0}\frac{1}{r^2}\left[\frac{\tilde{p}\cos\theta}{\Lambda}\right]_{\text{ret}}\mathrm{d}\Sigma \quad (22b)$$

where $\tilde{p} = p_a + (\gamma - 1)p_0$. Four other equivalent forms may be obtained by using Eq. (10). These are new forms of solution of the FW-H equation. These have not been used for numerical calculations but are presented here for their unusually simple forms.

In the application of Eq. (20) for blades with finite thickness at the tip, it was found that the sources on the airfoil section at the blade tip are very important.[15] The effect of these sources can be described by a line integral over the blade tip chord as shown in Ref. 15. This observation also applies to Eq. (22).

3) Another useful interpretation of thickness noise is in terms of the volume inside the surface $F=0$. The far field result of this interpretation was published earlier.[4] Here, it is extended to the near field also. In this paper the methods of Hawkings and Lowson[6] and Succi[16] are based on the results derived below.

The surface source term in Eq. (16) can be written as a volume source as follows. Let $H(f)$ be the Heaviside function. Then it is seen that

$$\frac{\partial}{\partial t}[1-H(f)] = v_n|\nabla f|\delta(f) \quad (23)$$

Equation (16) can be written as follows when the above result is used:

$$\square^2 p'_T = \frac{\partial^2}{\partial t^2}\{\rho_0[1-H(f)]\} \quad (24)$$

The solution of this equation, using Green's function of the wave operator, is

$$4\pi p'_T(x,t) = \frac{\partial^2}{\partial t^2}\int\frac{\rho_0[1-H(F)]}{r}\mathrm{d}y = \frac{\partial^2}{\partial t^2}\int_{F<0}\frac{\rho_0\mathrm{d}y}{r} \quad (25)$$

Note that the integration is over the volume inside the surface Σ: $F=0$. For the observer in the far field the above equation can be simplified as follows

$$4\pi p'_T(x,t) = \frac{\rho_0}{r}\frac{\partial^2}{\partial t^2}\int_{F<0}\mathrm{d}y \quad (26a)$$

$$=\frac{\rho_0}{r}\frac{\partial^2\Psi}{\partial t^2}\ \text{(far field)} \quad (26b)$$

Here $\Psi(x,t)$ is the volume inisde the surface Σ which depends on the observer position and time. It must be mentioned that the observer is in the far field if $r_{\min} \gg L$ where $r_{\min}$ is the minimum distance of the observer from the blade surface and L is the maximum dimension of the surface $F=0$.

4) Consider the surface $F(y;x,t) = [f(y,\tau)]_{\text{ret}} = 0$. The local normal velocity of this surface as a function of observer time is

$$V_N = -\frac{\partial F/\partial t}{|\nabla F|} = \left[\frac{v_n}{\Lambda}\right]_{\text{ret}} \quad (27)$$

Substituting this relation in the integral of Eq. (17a), one gets

$$4\pi p'_T(x,t) = \frac{\partial}{\partial t}\int_{F=0}\frac{\rho_0 V_N}{r}\mathrm{d}\Sigma \quad (28a)$$

$$=\frac{\partial}{\partial t}\int_{F=0}\frac{\mathrm{d}Q_0}{r} \quad (28b)$$

$$=\frac{1}{r}\frac{\partial Q_0}{\partial t}\ \text{(far field)} \quad (28c)$$

where $\mathrm{d}Q_0 = \rho_0 V_N \mathrm{d}\Sigma$ is the local rate of mass injection from the surface Σ. Clearly

$$Q_0 = \int_{F=0}\rho_0 V_N \mathrm{d}\Sigma \quad (29)$$

is the net rate of mass injection from this surface. Equations (28a-c) are just another interpretation of thickness noise. These are the noncompact source extension of the well-known result that for a stationary compact monopole source, the acoustic pressure is proportional to the time derivative of the rate of mass injection.

5) The following result applies to a rigid body in motion. Since it has a known compact limit formulation, it will be derived here. Assume that the velocity vector field inside the rigid body is described by the local velocity of the rigid body. This vector field is solenoidal, i.e., $\nabla\cdot v = 0$. Now consider the following manipulations:

$$\frac{\partial}{\partial x_i}\{v_i[1-H(f)]\} = \frac{\partial v_i}{\partial x_i}[1-H(f)] - v_i\frac{\partial f}{\partial x_i}\delta(f) = -v_n|\nabla f|\delta(f)$$

Using this result in Eqs. (16), the wave equation for the thickness noise can be written as

$$\square^2 p'_T = -\frac{\partial^2}{\partial t\partial x_i}\{\rho_0 v_i[1-H(f)]\} \quad (30)$$

A solution of this equation using Green's function approach is

$$4\pi p'_T(x,t) = -\frac{\partial^2}{\partial t\partial x_i}\int_{f<0}\left[\frac{\rho_0 v_i}{r|1-M_r|}\right]_{\text{ret}}\mathrm{d}y \quad (31)$$

Summarizing, it is seen that the thickness noise is the solution of the wave equation for p'_T with the following equivalent forms of the source term

$$\Box^2 p'_T = \frac{\partial^2}{\partial t^2}\{\rho_0[1-H(f)]\} \quad (32a)$$

$$= \frac{\partial^2}{\partial x_i^2}\{\rho_0 c^2[1-H(f)]\} \quad (32b)$$

$$= \frac{\partial}{\partial t}\{\rho_0 v_n |\nabla f|\delta(f)\} \quad (32c)$$

$$= -\frac{\partial}{\partial x_i}\{\rho_0 c^2 n_i |\nabla f|\delta(f)\} \quad (32d)$$

$$= -\frac{\partial^2}{\partial t \partial x_i}\{\rho_0 v_i[1-H(f)]\}\text{(rigid body)} \quad (32e)$$

Note that Eq. (32b) is equivalent to Eq. (32d) if the partial derivative $\partial/\partial x_i$ in the former equation is taken inside the curly bracket.

Compact Source Formulas

In the following paragraphs the compact source formulas are derived as the limit of noncompact source relations derived above. Although it is possible to write down the inhomogeneous terms of the acoustic wave equation for moving sources by other methods (for example, from the equations for stationary point sources), the method presented here is more rigorous. One can also establish conditions for compactness of the sources.

Loading Noise

Only one result will be presented here. The starting point of this analysis is the following equation:

$$4\pi p'_L(x,t) = -\frac{\partial}{\partial x_i}\int_{f=0}\left[\frac{l_i}{r|1-M_r|}\right]_{\text{ret}}\mathrm{d}S \quad (33)$$

For fixed x and t, let L be the maximum dimension for the Σ surface $F=0$. Let the minimum value of r denoted by $r_{\min}$ be such that $r_{\min} \gg L$. Let $\Delta\tau$ be the time taken by the collapsing sphere $g=0$ to cross the body. If $\tilde{T}$ is the typical period of fluctuation of forces on the medium and $\Delta\tau \ll \tilde{T}$, then one can simplify Eq. (33) as follows:

$$4\pi p'_L(x,t) = -\frac{\partial}{\partial x_i}\left[\frac{1}{r|1-M_r|}\int_{f=0} l_i \mathrm{d}S\right]_{\text{ret}}$$

$$= -\frac{\partial}{\partial x_i}\left[\frac{F_i(\tau)}{r|1-M_r|}\right]_{\text{ret}} \quad (34)$$

where $F_i(\tau)$ is now the net force of the body on the fluid medium. This result is due to Lowson.[17] It is the solution of the following equation:

$$\Box^2 p'_L = -\frac{\partial}{\partial x_i}\{F_i(t)\delta[x-s(t)]\} \quad (35)$$

from which Lowson obtained his near and far field results. Here $s(t)$ is the position vector of the point force.

Using Eq. (14), one can write another form of Eq. (34) as follows:

$$4\pi p'_L(x,t) = \frac{1}{c}\frac{\partial}{\partial t}\left[\frac{F_r}{r|1-M_r|}\right]_{\text{ret}} + \left[\frac{F_r}{r^2|1-M_r|}\right]_{\text{ret}} \quad (36a)$$

$$= \left[\frac{\dot{F}_r}{cr|1-M_r|^2}\right]_{\text{ret}} + \left[\frac{F_r \dot{M}_r}{r|1-M_r|^3}\right]_{\text{ret}}\text{(far field)} \quad (36b)$$

where $(\cdot)$ indicates the source time derivative and $F_r = F_i \hat{r}_i$ is the net force in the radiation direction. Succi's method[16] is based on Eq. (36a). Lowson has used the above results extensively in his work on rotating blade noise.

The conditions

$$r_{\min} \gg L \quad (37a)$$

$$\Delta\tau \ll \tilde{T} \quad (37b)$$

together determine the compactness of the source. Note that compactness is not solely a property of the source. It also depends on the observer position.

Thickness Noise

1) The above analysis can be applied to Eq. (25) assuming that $f=0$ is a rigid body. The derivation is as follows:

$$4\pi p'_T(x,t) = \rho_0 \frac{\partial^2}{\partial t^2}\int_{F<0}\frac{\mathrm{d}y}{r} = \rho_0 \frac{\partial^2}{\partial t^2}\int_{f<0}\left[\frac{1}{r|1-M_r|}\right]_{\text{ret}}\mathrm{d}y$$

$$= \rho_0 \frac{\partial^2}{\partial t^2}\left[\frac{1}{r|1-M_r|}\int_{f<0}\mathrm{d}y\right]_{\text{ret}}$$

$$= \rho_0 \frac{\partial^2}{\partial t^2}\left[\frac{\psi}{r|1-M_r|}\right]_{\text{ret}} \quad (38)$$

where ψ is the volume inside the actual body $f=0$. The same relation holds for a pulsating body with volume $\psi(\tau)$ which is in motion. The quantity M_r is then based on the velocity of the origin of the frame moving with the pulsating body. Equation (38) is the solution of the following wave equation:

$$\Box^2 p'_T = \frac{\partial^2}{\partial t^2}\{\rho_0 \psi \delta[x-s(t)]\} \quad (39)$$

where $s(t)$ is the source position. In the present paper, Succi's method is derived based on Eq. (38).

2) Applying the above analysis to the solution of Eq. (32b), one obtains another equivalent form of thickness noise

$$4\pi p'_T(x,t) = \frac{\partial^2}{\partial x_i^2}\left[\frac{\rho_0 c^2 \psi}{r|1-M_r|}\right]_{\text{ret}} \quad (40)$$

This is the solution of the wave equation

$$\Box^2 p'_T = \frac{\partial^2}{\partial x_i^2}\{\rho_0 c^2 \psi \delta[x-s(t)]\} \quad (41)$$

Alternatively, applying the wave operator to $\rho_0 c^2 \psi \delta[x-s(t)]$, one obtains

$$\Box^2\{\rho_0 c^2 \psi \delta[x-s(t)]\} = \frac{\partial^2}{\partial t^2}\{\rho_0 \psi \delta[x-s(t)]\} - \frac{\partial^2}{\partial x_i^2}\{\rho_0 c^2 \psi \delta[x-s(t)]\} \quad (42)$$

The solution of this equation is

$$4\pi \rho_0 c^2 \psi \delta[x-s(t)] = \frac{\partial^2}{\partial t^2}\left[\frac{\rho_0 \psi}{r|1-M_r|}\right]_{\text{ret}} - \frac{\partial^2}{\partial x_i^2}\left[\frac{\rho_0 c^2 \psi}{r|1-M_r|}\right]_{\text{ret}} \quad (43)$$

For points not coinciding with the source itself, one has $\delta[x-$

$s(t)]=0$. Therefore,

$$\frac{\partial^2}{\partial t^2}\left[\frac{\rho_0\psi}{r|1-M_r|}\right]_{\rm ret}=\frac{\partial^2}{\partial x_i^2}\left[\frac{\rho_0 c^2\psi}{r|1-M_r|}\right]_{\rm ret} \tag{44}$$

Equation (40) has not been used for noise calculations. It is the compact form of the solution of Eq. (32b).

3) From the noncompact solution of Eq. (32e) and conditions described by Eq. (37), another compact source formula for thickness noise is obtained as follows:

$$4\pi p_T'(x,t)=-\frac{\partial^2}{\partial t\partial x_i}\left[\frac{\rho_0 v_i}{r|1-M_r|}\right]_{\rm ret} \tag{45}$$

Here, $v_i=v_i(\tau)$ is the velocity of the source. This equation is the solution of the wave equation

$$\Box^2 p_T'=-\frac{\partial^2}{\partial t\partial x_i}\{\rho_0 v_i\psi\delta[x-s(t)]\} \tag{46}$$

Another method of deriving this equation is converting one of the time derivatives on the right side of Eq. (39) to a space derivative as shown below:

$$\frac{\partial}{\partial t}\delta[x-s(t)]=\frac{\partial}{\partial t}\{\delta[x_1-s_1(t)]\delta[x_2-s_2(t)]\delta[x_3-s_3(t)]\}$$

$$=-\frac{\partial}{\partial x_i}\left\{\frac{{\rm d}s_i}{{\rm d}t}\delta[x-s(t)]\right\}=-\frac{\partial}{\partial x_i}\{v_i\delta[x-s(t)]\} \tag{47}$$

The well-known work of Arnoldi[18] is based on the solution of Eq. (46).

Summarizing, the thickness noise for a compact source is the solution of the following wave equation with any of the three equivalent source functions:

$$\Box^2 p_T'=\frac{\partial^2}{\partial t^2}\{\rho_0\psi\delta[x-s(t)]\} \tag{48a}$$

$$=\frac{\partial^2}{\partial x_i^2}\{\rho_0 c^2\psi\delta[x-s(t)]\} \tag{48b}$$

$$=-\frac{\partial^2}{\partial t\partial x_i}\{\rho_0 v_i\psi\delta[x-s(t)]\} \tag{48c}$$

III. Current Formulations Used by Various Researchers

In the last decade, many researchers have developed computer programs to calculate the discrete frequency noise of rotating blades. In this section, the method used by these researchers will be discussed. Each method has certain merits and restrictions, some of which will be pointed out. As can be seen from the reference articles, one can algebraically manipulate and specialize the equations in the previous section considerably for noise calculation. It is neither useful nor possible to present all these formulas. The acoustic formulas in this paper are given in their most general form which may differ somewhat from those by the originators of the methods. This is done to reduce the number and length of formulas in this paper. The methods are discussed in chronological order of the first publication of each research worker.

The Method of Hawkings and Lowson

This paper appeared in 1974.[6] Their method is applicable to open subsonic or supersonic rotors which are not in forward motion (hovering rotor or static propeller). Since the acoustic pressure signature is periodic, the discrete frequency acoustic spectrum of the noise can be obtained by the conventional Fourier series expansion of the signature. The loading noise is considered first. Following Hawkings and Lowson, only far field results will be presented.

From Eq. (15c), the far field loading noise is described by

$$4\pi p_L'(x,t)=\frac{1}{c}\frac{\partial}{\partial t}\int\left[\frac{l_r}{r|1-M_r|}\right]_{\rm ret}{\rm d}S \tag{49}$$

The blades are assumed thin and in the x_1x_2 plane of a nonrotating x frame with origin at the center of the rotor disk. The observer is assumed in x_1x_3 plane at the distance of r_0 from the origin and making an angle θ_0 with the rotor axis (x_3 axis). Define the Fourier decomposition of p_L' as follows:

$$p_L'(x,t)=\sum_{-\infty}^{\infty}p_{Ln}'(x)e^{-in\omega t} \tag{50}$$

where $\omega=2\pi\bar{f}$. Here $\bar{f}$ is the shaft frequency. Only a single blade is considered now.

The following relation is used to find $p_{Ln}'(x)$

$$p_{Ln}'=\frac{1}{\bar{T}}\int_0^{\bar{T}}p_L'(x,t)e^{in\omega t}{\rm d}t$$

$$=-\frac{in\omega}{4\pi c r_0\bar{T}}\int_{\rm PF}\int_0^{\bar{T}}\left[\frac{l_r}{|1-M_r|}\right]_{\rm ret}e^{in\omega t}{\rm d}t{\rm d}S \tag{51}$$

where PF stands for planform of the blade. Note that an integration by parts was performed to get rid of the observer time derivative. With the above definition of p_{Ln}', to find the decibel level of nth harmonic, one must use the relation for positive integers n

$$L_n=20\log_{10}\left(\frac{\sqrt{2}|p_{Ln}'|}{2\times10^{-5}}\right)\quad(\text{re: }2\times10^{-5}\text{ Pa}) \tag{52}$$

Now, consider a source on the blade with polar coordinates (R,ϕ) in a frame fixed to the blade. The variable t is transformed to τ.

Then the following relations are used in Eq. (51):

$$t=\tau+(r/c)\approx\tau+(r_0/c)[1-(R/r_0)\sin\theta_0\cos\beta] \tag{53}$$

$${\rm d}t=|1-M_r|{\rm d}\tau \tag{54}$$

$$\mathcal{Z}_r=-T\cos\theta_0-D\sin\theta_0\sin\beta \tag{55}$$

where $\beta=\omega\tau+\phi$ is the azimuthal angle of the source in the x frame. Also, T and D are thrust (or lift) and drag per unit area at the source point on the blade. Integration of Eq. (52) with respect to τ then gives

$$p_{Ln}'=\frac{in\omega}{4\pi c r_0}\exp\left[in\left(\frac{\omega r_0}{c}-\frac{\pi}{2}\right)\right]$$

$$\times\int_{\rm PF}\left(T\cos\theta_0-\frac{D}{M}\right)J_n(nM\sin\theta_0)\exp(-in\phi)R{\rm d}R{\rm d}\phi \tag{56}$$

where $M=R\omega/c$ is the Mach number of the source. The integration is over the blade planform.

For the thickness noise, it is better to start with Eq. (26b). The volume

$$\Psi=\int_{F<0}{\rm d}y$$

can be written as integral over the planform as follows

$$\Psi=\int_{\rm PF}\left[\frac{h(R,\phi)}{|1-M_r|}\right]_{\rm ret}R{\rm d}R{\rm d}\phi \tag{57}$$

where $h(R,\phi)$ is the blade thickness. The same analysis leading to Eq. (56) can be repeated to give the Fourier component of the thickness noise:

$$p'_{Tn} = \frac{\rho_0 (in\omega)^2}{4\pi r_0} \exp\left[in\left(\frac{\omega r_0}{c} - \frac{\pi}{2}\right)\right]$$

$$\times \int_{PF} hJ_n(nM\sin\theta_0)\exp(-in\phi)R\mathrm{d}R\mathrm{d}\phi \qquad (58)$$

For B blades, n on the right-hand side of Eq. (56) and Eq. (58) must be replaced by nB and the resulting right-hand side of the equations must be further multiplied by B. Note that Eqs. (56) and (58) differ by a factor of two in the denominator from those of Hawkings and Lowson. This factor appears because the definition of Fourier series expansion in the present paper, Eq. (50), differs from that of Hawkings and Lowson.

There is no restriction on tip Mach number of the blades. The main restrictions of this method are 1) far field position of the observer; 2) no forward motion of the rotor is allowed; and 3) the blade's mean chord surface is approximately in the plane of the rotor disk.

The Method of Farassat

This method was originally based on Eqs. (15b) and (17b).[8,19] Both the near and far field noise could be calculated. The blade mean chord surface was assumed to be in the plane of the disk. In a more recent version of this method, each blade is specified precisely in curvilinear coordinates fixed to the blade. The blades are subdivided into panels. The collapsing sphere method, based on Eqs. (15b) and (17b) is used for each panel for which M_r at the emission time is greater than 0.98 and Eqs. (15c) and (17c) are used if $M_r < 0.98$. There are no restrictions on the blade shape, tip Mach number, or forward motion of the blades. This method was developed in 1978 to calculate the noise of a prop-fan. For details of the implementation of this method, see Refs. 9 and 10. In this method, all acoustic calculations are in the time domain. A similar computer program for helicopter rotors with subsonic tip Mach numbers based on Eq. (15c) and (17c) was developed in 1979 at NASA Langley by Nystrom and Farassat.

The Methods of Hanson

Hanson has two methods, one in the time domain[7] and another in the frequency domain.[20] The method in the time domain is based on Eqs. (15a) and (17a). The sources are distributed on the mean chord surface of the blades so that the surface Σ is generated by the intersection of the collapsing sphere $g=0$ and the blade planform. This surface is called the acoustic planform. Since $M_n = 0$ for this surface (infinitely thin), then $\Lambda = 1$. The force in the radiation direction per unit area becomes $-\Delta p\cos\theta$ where Δp is the local lift per unit area on the blade and θ is the angle between the normal to the mean chord surface of the blade (pointing to the suction side) and the radiation direction. The equation used by Hanson and the time domain calculation, in the notation of the present paper is:

$$4\pi p'(\boldsymbol{x},t) = \frac{1}{c}\frac{\partial}{\partial t}\int_{APF} \frac{1}{r}[2\rho_0 c v_n - \Delta p\cos\theta]_{\text{ret}}\mathrm{d}\Sigma$$

$$- \int_{APF} \frac{1}{r^2}[\Delta p\cos\theta]_{\text{ret}}\mathrm{d}\Sigma \qquad (59)$$

where APF stands for the acoustic planform. In Hanson's work,[7] $\mathrm{d}\Sigma$ is written explicitly in terms of variables defining a helical surface on which the sources lie. Note that the negative sign for the loading noise term above appears because of the assumption that the normal to the planform points to the suction side of the blade. Also, v_n on one side of the blade is used in Eq. (59).

Hanson's frequency domain result, which was developed for propellers in forward flight, is derived by a Fourier transform technique.[20] The far field result for a single blade will be presented. Let a rotating locally orthogonal frame fixed to the blade, denoted as the $\boldsymbol{\eta}$ frame, be set up as follows. The η_2 axis is along the pitch change axis, the η_1 axis is parallel to the chord and the η_3 axis is such that the $\boldsymbol{\eta}$ frame forms a right-handed coordinate system. If the source term on the right side of the FW-H equation is denoted by $Q(\boldsymbol{\eta})$, then the Fourier transform $\tilde{Q}(\lambda/U, \eta_2, \eta_3)$ of Q is defined as follows:

$$\tilde{Q}\left(\frac{\lambda}{U},\eta_2,\eta_3\right) = \int_{-\infty}^{\infty} Q(\boldsymbol{\eta})e^{i(\lambda/U)\eta_1}\mathrm{d}\eta_1 \qquad (60)$$

where $U^2 = (\eta_2\omega)^2 + V_F^2$ is the square of the local helical speed and V_F is the forward velocity of the propeller. The angular velocity of the propeller is denoted by ω. A new shifted frequency is defined by the relation

$$\omega_D = \frac{\omega}{1 - M_F\cos\theta_0} \qquad (61)$$

where $M_F = V_F/c$ and θ_0 is the angle between the propeller axis and the observer direction. The observer distance from the propeller center is assumed as r_0. Then the complex amplitude of the nth harmonic of the acoustic pressure spectrum is given by

$$p'_n(\boldsymbol{x}) = \frac{\omega_D}{4\pi\omega r_0}\exp\left[in\left(\frac{\omega_D r_0}{c} - \frac{\pi}{2}\right)\right]$$

$$\times \int_0^{\infty} J_n(nM_D\sin\theta_0)\int_{-\infty}^{\infty}\tilde{Q}\left(\frac{n\omega_D}{U},\eta_2,\eta_3\right)$$

$$\times \exp\left[i\frac{n}{U}\left(\omega M_D\cos\theta_0 - \frac{V_F}{r_0}\right)\eta_3\right]\mathrm{d}\eta_3\mathrm{d}\eta_2 \qquad (62)$$

where $M_D = \omega_D\eta_2/c$. The modification of this equation for n blades is obvious.

The above equation has been used extensively by Hanson for prop-fan noise calculations. Although this result is developed for the far field, it has been used successfully for aeroacoustic optimization of prop-fans in the design stage using a phasor method as employed in electrical engineering.[20] Generally good agreement between the methods of Hanson and Farassat has been obtained for several prop-fan designs.[10,20]

The Method of Woan and Gregorek

This method is based on Eqs. (15c) and (17c) with the derivative $\partial/\partial t$ taken inside the integrals.[11] It is assumed that the blade forces are steady. The result is as follows:

$$4\pi p'(\boldsymbol{x},t)$$

$$= \int_{f=0}\left[\frac{1}{c|1-M_r|}\frac{\partial}{\partial\tau}\left(\frac{\rho_0 c v_n + l_r}{r|1-M_r|}\right) + \frac{l_r}{r^2|1-M_r|}\right]_{\text{ret}}\mathrm{d}s \qquad (63)$$

In this equation r, M_r and l_r are functions of time. Then the following relations are used in the above equation

$$\frac{\partial r}{\partial\tau} = -v_r \qquad (64)$$

$$\frac{\partial\hat{r}_i}{\partial\tau} = \frac{\hat{r}_i v_r - v_i}{r} \qquad (65)$$

$$\frac{\partial M_r}{\partial\tau} = \frac{1}{cr}\left[r_i\frac{\partial v_i}{\partial\tau} + v_r^2 - v^2\right] \qquad (66)$$

where $v_r = v_i \hat{r}_i$ and $v^2 = v_i v_i$. Note that l_i is also a function of τ.

Woan and Gregorek use a curvilinear coordinate system to describe the blade and use Gauss' formula for the element of surface area of the blade. The only restriction is that the tip speed of the blade must be subsonic. The computer program developed based on this method is for propellers in forward flight.[11] This method is suitable for helicopter rotor noise calculations.

The Method of Succi

In this method, the blade is divided into small segments. Each segment has a finite volume ψ_k and a force acting on the fluid L_{ki}. The forces acting on the fluid are assumed steady, although this restriction can be easily relaxed. The noise of the blades is calculated by summing over all the segments using Eqs. (36a) and (38):

$$4\pi p'(x,t) = \sum_k \left[\frac{1}{c} \frac{1}{|1-M_r|} \frac{\partial}{\partial \tau} \left(\frac{L_{ki}\hat{r}_i}{r|1-M_r|} \right) + \frac{L_{ki}\hat{r}_i}{r^2 |1-M_r|} \right]_{\text{ret}} + \sum_k \left[\frac{\rho_0 \psi_k}{|1-M_r|} \frac{\partial}{\partial \tau} \left(\frac{1}{|1-M_r|} \frac{\partial}{\partial \tau} \left(\frac{1}{r|1-M_r|} \right) \right) \right]_{\text{ret}} \tag{67}$$

Equations (64) through (66) are used to get the final expression for use on a computer. The main advantages for this method are the ease of computer coding and the speed of execution. The current use of this program is for propellers in forward flight with subsonic tip speed.[9,16] This method is also suitable for helicopter rotor noise calculations.

The Method of Jou

This method is an extension of the Hawkings and Lowson method.[6] The extension is similar to the technique of Garrick and Watkins where the forward flight effect was included in Gutin's result.[22] The analytic results of Jou were developed for a propeller in uniform forward flight with constant speed V_F. The sources are assumed to lie in the propeller disk. The acoustic sources on the blade can be replaced by equivalent periodic sources on the entire propeller disk. By a simple geometric construction (sometimes referred to as the Garrick triangle), the actual sound transmission distance can be related to the observer coordinates in the frame moving with the propeller. In the notation of the present paper, the complex amplitude of the nth harmonic (for a one-bladed propeller) of the loading noise in the far field is

$$p'_{Ln}(x) = \frac{in\omega}{4\pi c k_D r_0} \exp\left\{ in \left[\frac{\omega r_0 k_D}{c(1-M_F^2)} - \frac{\pi}{2} \right] \right\} \times \int_{\text{PF}} \left[T\left(\frac{\cos\theta_0}{(1-M_F^2 \sin^2\theta_0)^{1/2}} - M_F \right) - D \frac{1-M_F^2}{M} \right] \times \frac{J_n(nY)}{\cos\sigma} \exp(-in\phi) R \mathrm{d}R \mathrm{d}\phi \tag{68}$$

where the following definitions for the symbols are used:

$$k_D = (1-M_F^2 \sin^2\theta_0)^{1/2} - M\cos\theta_0 \tag{69}$$

$$Y = \frac{M \sin\theta_0}{(1-M_F^2 \sin^2\theta_0)^{1/2}} \tag{70}$$

Here $M = R\omega/c$ is the Mach number at radial distance R and σ is the twist angle (geometric angle of attack).

For the thickness noise, the complex amplitude of the nth harmonic in the far field is

$$p'_{Tn}(x) = \frac{in\omega\rho_0 c}{4\pi k_D r_0} \exp\left\{ in \left[\frac{\omega r_0 k_D}{c(1-M_F^2)} - \frac{\pi}{2} \right] \right\} \times \int_{\text{PF}} U \frac{\partial h}{\partial s'} (1-M_F^2) \frac{J_n(nY)}{\cos\sigma} \exp(-in\phi) R \mathrm{d}R \mathrm{d}\phi \tag{71}$$

where $U^2 = R^2\omega^2 + V_F^2$ is the square of the local helical speed and s' is the distance along the direction of the helical velocity vector.

The above equations have not been used for numerical calculations of propeller noise. Some interesting qualitative results have been published by Jou concerning the directivity of high speed propeller noise.

IV. Concluding Remarks

In this paper, many of the formulas used in calculation of the noise of rotating blades are presented. A unified approach based on the solution of the FW-H equation is used. An interesting observation is the diversity of the forms of the solution of the linearized wave equation derived by the workers in the field.

Most equations presented in this paper have been used for noise calculation. In the computer programs developed for this purpose, various approximations are made to reduce execution time. An important unanswered question is how these approximations influence the numerical results. An obviously good policy is to use as few approximations as possible in the coding.

It is not possible to select one particular formulation which can be used for all rotating blade noise problems. For example, although the collapsing sphere method is the main choice in time domain for high speed rotating blades (transonic and supersonic tip speeds), it is not recommended for blades with subsonic tip speed. In this latter case, the formulation with the Doppler factor is more efficient. Since in the aeroacoustic optimization of rotating blades, one may be required to control the level of some harmonics of the acoustic spectrum, the frequency domain methods are particularly suitable for this task.

A careful study of the available formulations as well as other factors such as the type of motion of the blade disk (as a propeller or a helicopter rotor) and observer position (near or far field) is required in development of a computer program for noise calculation.

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